Electromechanical modeling and design for phase-control of locked modes in DIII-D

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Outline of presentation

1. Motivation
2. Overview and purpose of control-oriented model
3. Examples of model properties
4. DIII-D phase-control design: external 3D coils
5. Integrated phase-control simulation
6. Conclusion
Locked-modes may be more efficiently and reliably suppressed with the aid of a phase-controller

- Locked 2/1 modes degrade confinement and may lead to disruptions
  - Suppression of mode before locking may fail

- It is possible to suppress locked 2/1 modes with ECCD + RMPs
  - Ultimate ambition: automated and robust H-mode recovery

- The locked-mode O-point phase may be misaligned to ECCD
  - Reasons include residual error-field and viscous torque
  - Effects may be time-varying and unpredictable

- Feedback control of 2/1 phase may make alignment to ECCD reliable
  - Prioritize a design using external coils (reactor relevance)
Design of a basic phase-controller can be guided by first modeling the electromechanics of mode-wall-coils

- Dynamical systems model: not a torque-balance condition

- Focus on (nearly) locked-mode aspects: frequency < several kHz
  - Thin wall approximation (skin depth>thickness) mostly OK

- Maintain simplicity, aid physics insight, fast simulation
  - Control-oriented modeling
  - Feedback may handle unmodeled and uncertain effects
  - Small ODE, lumped parameters
Nonlinear system model key components are (i) thin resistive wall, (ii) coils, (iii) mode with inertia

- Model structure is obtained from a linear periodic geometry

- But it is calibrated based on
  - Axisymmetric plasma equilibrium
  - Machine geometry incl. 3D coil description
Additional inputs include error-field, a generic torque, and prescribed NTM width evolution.

e.g. NBI, viscous effects

- Error field
- Aux. torque
- $dW/dt$
- NTM size, phase, velocity
- Nonlinear evolution
- Resistive thin wall current
- Internal coils

"rotor"

"stator"

"stator"

Linear subsystem

"stator"

"stator"

"stator"

"stator"

"stator"

"stator"

"stator"

"stator"
Purpose is to design a feedback to the external coils to reject the effect of perturbation signals on the phase.
Purpose is to design a feedback to the external coils to reject the effect of perturbation signals on the phase with margin for parameter errors (e.g. density, coupling).
Basic form of model is a small set of nonlinear ordinary differential equations for a particular \((m,n)\):

Space-vector (Fourier space) formulation:

\[
\begin{align*}
\tau \dot{x}_1 + x_1 &= \alpha_s x_4 + \alpha_b J_1^b + \alpha_c J_1^c \\
\tau \dot{x}_2 + x_2 &= \alpha_s x_5 + \alpha_b J_2^b + \alpha_c J_2^c \\
\dot{x}_3 &= \beta_1 \left( y_1 x_5 - y_2 x_4 \right) \\
\dot{x}_4 &= n x_3 x_5 \\
\dot{x}_5 &= -n x_3 x_4
\end{align*}
\]

\[
(y_1, y_2) = \gamma_1(x_1, x_2) + \gamma_2(J_1^b, J_2^b)
\]

\[
\beta_1 = n/(2 \mu_0 R^2 W \rho)
\]

\[|J_s| = \sqrt{x_4^2 + x_5^2} = C_1 W^2\]

\[|B_w| = \sqrt{x_1^2 + x_2^2}\]

\(\tau\) thin-wall time constant (~3ms for 2/1)
In the absence of coil inputs, error-field and torque, the classical induction motoring/braking curve is obtained.

- This is a **steady-state** property
- Black squares: VALEN calculation (D. Shiraki)

\[ T(\omega) \propto \frac{(n \omega \tau)}{\left(1 + (n \omega \tau)^2\right)} \]

#126623 @ 3000 ms; \( \tau_w = 2.91 \) ms; \( f_{\text{peak}} = 54.7 \) Hz

\( m/n = 2/1 \)
Mode-locking simulations exhibits a transient effect as a result of the low mechanical inertia and strong currents

- Induction braking of mode spinning at several kHz
- Slow-down time (2 kHz to zero) is consistent with DIII-D observations
  - *DB analysis: 15 ± 10 ms (Sweeney & Choi)*
- Locking-oscillation appears (no viscous damping effect included)

**Ringing condition:** \(2 \omega_\infty \tau > 1\)

Here: 2 kHz to 0 Hz in ~17 ms. No preferred direction without EF or coils.
Open-loop locked-mode entrainment using internal or external coils: stability assessment in synchronous frame

\[ \tau \dot{B}_w = -B_w + \alpha_s J_s \cos(\varphi_2 - \varphi_1) + \alpha_c J_c \cos(\varphi_1) \]
\[ \tau B_w \dot{\varphi}_1 = -\tau B_w \omega_c + \alpha_s J_s \sin(\varphi_2 - \varphi_1) - \alpha_c J_c \sin(\varphi_1) \]
\[ \ddot{\varphi}_2 = -n \beta_1 \gamma_1 B_w J_s \sin(\varphi_2 - \varphi_1) \]

1. Find fixed-points (torque balance)
2. Evaluate Jacobian and its stability of points 1
Open-loop entrainment stability charts display maximum stable rotation frequencies.

Critical entrainment frequency [Hz]

$W = 7.5\text{cm}$

$I_C = 3\text{ kA}$

$f_{\text{max}} \sim 121\text{ Hz}$
Comparable open-loop entrainment stability for both I-coils and C-coils

**C-coils**

Critical entrainment frequency [Hz]

<table>
<thead>
<tr>
<th>NTM width W [cm]</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-coil current I [kA]</td>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
<td>2.5</td>
<td>3.0</td>
<td>3.5</td>
<td>4.0</td>
<td>4.5</td>
<td>5.0</td>
</tr>
</tbody>
</table>

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**I-coils**

Critical entrainment frequency [Hz]

<table>
<thead>
<tr>
<th>NTM width W [cm]</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-coil current I [kA]</td>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
<td>2.5</td>
<td>3.0</td>
<td>3.5</td>
<td>4.0</td>
<td>4.5</td>
<td>5.0</td>
</tr>
</tbody>
</table>
Phase-control approach is based on linearizations of error-field perturbed static torque-balances with C-coils

(a) Set a magnitude for C-coils

(b) Set a magnitude of the EF

(c) Parametrize torque-balance in differential angle (a)-(b)

(d) Specify a range of mode sizes

(e) Linearize system dynamics for the parameters (c)-(d)

\[ \Delta(\xi) = \arctan \left( \frac{\epsilon_{EF} \sin(\xi)}{1 + \epsilon_{EF} \cos(\xi)} \right) \]

\[ 0 \leq \epsilon_{EF} = \frac{|B_{EF}|}{|(B_c)|} < 1 \]
The family of SISO* linear systems transfers perturbed C-coil angle to perturbed mode angle

\[ \text{input } u = \delta \varphi_c \quad \text{output } y = \delta \varphi \]

- System family exhibits a variable resonance
- Stay away from it (highly uncertain phase relations)
- Filtered proportional-integral controller \( C(s) \) suitable

\[ C(s) = K \left( 1 + \frac{1}{\tau_i s} \right) \frac{1}{\tau_f s + 1} \]

\( (*) \) SISO = single-input single-output
The family of SISO linear systems transfers perturbed C-coil angle to perturbed mode angle

input $u = \delta \phi_c$  

output $y = \delta \phi$

$\log_{10}(\omega)$ [rad/s]

$C(s) = K \left(1 + \frac{1}{\tau_i s}\right) \frac{1}{\tau_f s + 1}$

System family exhibits a variable resonance

Stay away from it (highly uncertain phase relations)

Filtered proportional-integral controller $C(s)$ suitable

One set of gains to control them all?
Formalize a proportional-integral-filter optimization as max-sensitivity-constrained min-max programming

Task is to maximize speed of disturbance rejection subject to gain- and phase margins:

\[ \min_{\tau_i, \tau_f} \max_{\theta \in \Theta} \left\{ \left( \frac{1}{T} \right) \int_0^T \| e(t) \| dt \right\} \quad T \gg \tau \\
max_{\theta \in \Theta} \| 1/(1+PC) \|_\infty \leq M_S = 1.4 \\
max_{\theta \in \Theta} \| (PC)/(1+PC) \|_\infty \leq M_T = 1.4 \\
\]

where \( d(t) \) is a load disturbance step at \( t=0 \)
A proportional-integral-filter solution with reasonable and apparently realistic performance can be found

\[
\min_{K, \tau_i, \tau_f} \max_{\theta \in \Theta} \left\{ \left( \frac{1}{T} \right) \int_0^T |e(t)| \, dt \right\} \quad T \gg \tau
\]

\[
\max_{\theta \in \Theta} \| \frac{1}{1 + PC} \|_\infty \leq M_S = 1.4
\]

\[
\max_{\theta \in \Theta} \| \frac{PC}{1 + PC} \|_\infty \leq M_T = 1.4
\]

Solved:

With \( d(t) \) a load disturbance step at \( t=0 \)

\[
I_C = 3 \text{ kA}; \ \xi = 0..\pi, \ \beta_{EF} = 1 \text{ G}
\]

\[
K = 0.719, \tau_i = 4.63 \text{ ms}, \ \tau_f = 2 \text{ ms}
\]
Nonlinear system simulation implemented in *simulink*

**Mode width evolution**
- 1
- dW/dt
- 1

**Auxiliary torque**
- 2
- T\_aux
- 2

**Error field**
- 3
- EF
- 3

**I-coil quartets**
- IU30; IU90; IU150
- 4
- In4
- 3

**C-coil difference pairs**
- C79, C139, C199
- 5
- In5
- 3

**System model:**
- coil-wall-NTM
- msfun\_wantm\_v0
- 7
- state
- 7

1
Out1
Nonlinear system simulation implemented in *simulink* to test start-up / triggering, and disturbance rejection.
Test scenario involves (a) NBI-like torque injection (b) mode-growth (c) background error-field & (d) I-coil toggle

(a) -1.5 Nm  
(b) +1.3 cm island size change  
(c) 0.8 G magnitude residual error field  
(d) 500 Amp. I-coil toggle
Integrated simulations of nonlinear system using triggered external coil feedback show encouraging results.
Animations: Mode, Coils, Wall, Ref. phasor

Mode locking & triggering

Stepping of reference phase

http://youtu.be/I9sdJIaApel

http://youtu.be/yallJG5p3IM
Animations: Mode, Coils, Wall, Ref. phasor

Rotating ref. 10 Hz

http://youtu.be/n3bFYEvQ2pI
Summary and conclusion.

- A basic model of mode-wall-coil dynamics was developed
  - Few “moving parts”

- The model was used to guide feedback control design for phase-alignment of the locked-mode
  - Not relying on accurate knowledge of parameters & states
  - No excessive bandwidth requirements

- Integrated simulations of model, triggered control, and disturbances show encouraging results