Abstract

A database has been developed to study the evolution, the nonlinear effects on equilibria, and the disruptivity of locked and quasi-stationary modes with poloidal and toroidal mode numbers $m=2$ and $n=1$ at DIII-D. The analysis of 22,500 discharges shows that more than 18% of disruptions are due to locked or quasi-stationary modes with rotating precursors (not including born locked modes). A locked mode stability parameter, formulated by the plasma internal inductance $l_i$ divided by the safety factor at 95% of the toroidal flux $q_{95}$, is found to exhibit significant predictive capability over whether a locked mode will disrupt or not, and does so up to hundreds of milliseconds before the disruption. Within 20 ms of the disruption, a parameter that measures the shortest distance between the island separatrix and the unperturbed last closed flux surface, referred to as $d_{edge}$, might surpass $l_i/q_{95}$ in its ability to discriminate disruptive and non-disruptive locked modes. $d_{edge}$ is also seen to correlate with the duration of the locked mode better than any other parameter that was considered. Within 50 ms of a locked mode disruption, average behavior shows exponential growth of the $n=1$ perturbed field, which might indicate exponential growth of the $m/n=2/1$ locked mode. Surprisingly, even assuming the aforementioned growth, disruptivity shows little dependence on island width up to 20 ms before the disruption. Timescales associated with the mode evolution are also studied and inform necessary response times for disruption avoidance and mitigation techniques. Observations of the evolution of $\beta_N$ during a locked mode, the effects of poloidal beta on the saturated width, and the reduction in Shafranov shift during locking are also presented.
(Motivation) Reactor tokamaks have to run without disruptions

1. Locked modes are a significant cause of disruptions in many tokamaks [DIII-D, JET, ITER?]

2. Confidence in locked mode disruption avoidance relies on understanding the physics of locked mode disruptions

3. To prevent locked mode disruptions, we can:
   1. Avoid locked modes (reduce field errors, eliminate seeds)
   2. Operate where locked modes are unlikely to disrupt
   3. Understand disruption physics well enough to drive plasma out of disruptive state (e.g. ECCD in island O-point, $I_p$ ramp-downs, others?)

*Sub-bullets 3.2 and 3.3 are the focus of this work*
Definitions

- **IRLM =** Initially Rotating Locked Mode (or locked mode with rotating precursor)
- **Disruptive IRLM =** IRLM that immediately precedes a disruption
- **Non-disruptive IRLM =** IRLM that decays or spins up prior to the $I_p$ ramp-down

**IRLM Disruptivity** = \[ \frac{\text{Number of disruptive IRLMs}}{\text{Number of IRLMs}} \]
Example of an initially rotating locked mode (IRLM)

1. $m/n = 2/1$ rotating mode
   - Source of rotating mode not studied

2. Mode locks
   1. Time between $f = 2$ kHz and locked referred to as *slow-down time*

3. Exists as locked mode
   1. Few to thousands of milliseconds
   2. Referred to as *survival time* for disruptive IRLMs

4. Disrupts or ceases to be a locked mode (decays or spins up)
Disruptions and IRLM global statistics
More than a quarter of high $\beta_N$ disruptions are due to IRLMs

- Study performed on shots 122000 to 159837 (2005 to 2014)
- 28% of all disruptions in shots with peak $\beta_N > 1.5$ are due to IRLMs, compared with 18% for all peak $\beta_N$
- Born locked modes not considered in this work
In a 1D study, IRLMs appear most often in intermediate $\beta_N$ plasmas

- Low occurrence at $\beta_N > 4.5$ might be explained by observed conditions of $q_{95} > 7$ and $T_{NBI} > 6$ NM in most of these shots

- 3D study of IRLM rate of occurrence vs. $\beta_N$, $T_{NBI}$, and $\rho_{q2}$ might be more informative
66% of 2/1 NTMs rotating at 2 kHz will lock in 45 ± 10 ms

- Indication of time available to prevent locking
- Larger $T_{wall}$ results in shorter slow-down time
Disruptive IRLMs most frequently survive 270 ± 60 ms

- 66% of disruptive modes terminate between 150 to 1010 ms
- Time available to avoid disruption after locking
Degradation of $\beta_N$ throughout IRLM evolution
On average, $\beta_N$ continually decreases throughout IRLM lifetime

- **(Left) $\beta_N$ tends to decrease between rotating onset and locking**
  - Purple – preceded by former locked mode

- **(Middle) Disruptive IRLMs decrease $\beta_N$ by up to 80%**

- **(Right) $\beta_N$ decreases by a smaller amount during lifetime of non-disruptive IRLMs**
IRLMs change the 2D equilibrium shape by reducing the Shafranov shift

- $\Delta R_0 / \Delta \beta_p \approx 4$ cm
- Decrease of $m/n=1/0$ shaping might affect toroidal coupling of $m/n=2/1$ with other $n=1$ perturbations
Decreasing IRLM disruptivity at high $\beta_N$ observed in 1D, partially attributed to coincident high $q_{95}$

- (Left) 1D projection of IRLM disruptivity vs. $\beta_N$ shows decreasing disruptivity with increasing $\beta_N$
- (Middle) IRLM disruptivity decreases with increasing $q_{95}$
- (Right) Percent distributions in $q_{95}$ show high betaN bins (purple and green) have less low $q_{95}$ discharges
IRLM amplitude and angular position behavior
From rotation at 2 kHz to 50 ms post locking, the island width usually does not change within error.

- Island widths not validated to better than ±2 cm (conservative error bar)
- ~30 small rotating islands grow significantly
Cumulative distribution of angular positions during locked, and quasi-stationary phase

Peaks offset from intrinsic EF by -35°, which is in the normal plasma rotation direction

Related to residual EF or viscous drag?
Saturated width scales with $\beta_p/(dq/dr)$, indicating at least partial drive from bootstrap deficit

- **Expected from steady–state Modified Rutherford Equation**

- **IRLMs occurring at low $q_{95}$ (top) correlate better than those at high $q_{95}$ (bottom)**

![Graph showing saturation width vs. $\beta_p/(dq/dr)$ at saturation](image)
Prior to disruption, IRLM might grow exponentially ($n=1$ field does)

- **(Left)** Five shots with IRLMs show a general trend of increasing $n=1$ field within 100 ms of disruption
- **(Middle)** Distributions of $n=1$ field shift toward higher field as disruption is approached
- **(Right)** Median of distributions grows exponentially in last 50 ms
- Poloidal analysis necessary to conclude growth of $m/n=2/1$
IRLM disruptivity scales strongly with $\rho_{q2}$ (at fixed $q_{95}$), and weakly with $q_{95}$ (at fixed $\rho_{q2}$)

- A 1D study (blue histograms) suggests both $\rho_{q2}$ and $q_{95}$ are important
- Fixing $\rho_{q2}$ shows that $q_{95}$ alone has a weak effect on IRLM disruptivity
- Fixing $q_{95}$ shows that $\rho_{q2}$ has a strong effect on IRLM disruptivity
- Are there any hidden variables here?
In DIII-D operational space, $\rho_{q2}$ is related to $l_i$ and $q_{95}$

$$\frac{l_i}{q_{95}} = \alpha \rho_{q2} + c$$

where $\alpha = 0.67 \pm 0.01$ and $c = -0.23 \pm 0.01$

- Empirically, find “extra high” correlation of $r_c = 0.87$
- What is the effect of $l_i$ in the previous study?
- Does $l_i$, $q_{95}$, or $\rho_{q2}(l_i/q_{95})$ determine IRLM disruptivity?
  - We will study how well a single parameter separates disruptive from non-disruptive IRLMs
Panels (24) and (25) inform investigation of $l_i/q_{95}$, and we find it performs better than $\rho_{q2}$.

- Separation between disruptive and non-disruptive IRLMs is predominantly vertical.
- Also, high correlation observed between $l_i/q_{95}$ and $\rho_{q2}$.

DIII-D ITER baseline
For two discrete probability distributions \( p \) and \( q \) parameterized by \( x \), over the parameter space \( X \), the BC value is given by,

\[
BC = \sum_{x \in X} \sqrt{p(x)q(x)}
\]

- \( BC=0 \) means \( p \) and \( q \) are completely separate
- \( BC=1 \) means \( p \) and \( q \) are identical (completely overlapping)

\(^1\)D. Comaniciu et al 2000 Comput. Vision Graph. 2
Of the parameters considered, $d_{\text{edge}}$ and $l_i/q_{95}$ separate disruptive from non-disruptive IRLMs best

Nearly indistinguishable
Assuming exponential growth of 2/1 mode, $d_{\text{edge}}$ becomes predictive within 20 ms of disruption.

$$d_{\text{edge}} = a - (r_{q2} + \frac{w}{2})$$

- $a$ plasma minor radius
- $r_{q2}$ minor radius of the $q=2$ surface
- $w$ island width

**Late predictive capability might suggest $d_{\text{edge}}$ is relevant to thermal quench physics**

**Even assuming exponential island growth, island width still hardly affects IRLM disruptivity (blue histogram)**
Parameters predictive of IRLM disruptions
Performance of $l_i/q_{95}$ and $d_{\text{edge}}$ can alternatively be quantified by percent missed disruptions and false alarms

<table>
<thead>
<tr>
<th>Parameter threshold</th>
<th>% missed disruptions</th>
<th>% false alarms</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_i/q_{95} &lt; 0.28$</td>
<td>18</td>
<td>10</td>
</tr>
<tr>
<td>$l_i/q_{95} &lt; 0.25$</td>
<td>11</td>
<td>15</td>
</tr>
<tr>
<td>$d_{\text{edge}} &gt; 9$ cm</td>
<td>20</td>
<td>12</td>
</tr>
<tr>
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<td>12</td>
<td>20</td>
</tr>
</tbody>
</table>

- $l_i/q_{95}$ also provides at least 100 ms of disruption warning, whereas $d_{\text{edge}}$ only provides 20 ms

- Although $d_{\text{edge}}$ is less favorable for disruption prediction, it might still be relevant to the physics of the disruption
Conclusions

1. In DIII-D, 18% of disruptions are due to IRLMs, and this percentage rises to 28% for $\beta_N > 1.5$
2. The survival time is most often 270 ms, and exhibits moderate correlation with $d_{\text{edge}}$
3. Disruptive IRLMs degrade $\beta_N$ by 50-80%, while non-disruptive IRLMs degrade $\beta_N$ less
4. Scaling of $w/r$ with $\beta_p/(dq/dr)$ suggests that IRLMs in DIII-D are driven at least in part by bootstrap deficit
5. The $n=1$ radial field grows exponentially within 50 ms of the disruption, and might imply exponential growth of $m/n=2/1$
6. $l_i/q_{95}$ is found to be predictive of IRLM disruptions 100 ms prior
   – $l_i/q_{95} < 0.25$ yields 11% missed disruptions and 15% false alarms
7. Assuming exponential growth of $m/n=2/1$, $d_{\text{edge}}$ becomes predictive within 20 ms of the disruption
APPENDIX
Disruption Identification
Plasma current decay time used to identify disruptions

- Decay time is the duration of the current quench\(^1\)

\[
t_d = \frac{\text{time of 80% } I_p - \text{time of 20% } I_p}{0.6}
\]

\(^1\)T.C. Hender 2010 IAEA Proceedings

Figure: D. Shiraki et al. 2015 NF
Three groupings

i. $t_d < 40 \text{ ms}$ \hspace{1cm} \textbf{Disruptive}

ii. $40 \text{ ms} < t_d < 200 \text{ ms}$ \hspace{1cm} \textbf{Intermediate}

iii. $t_d > 200 \text{ ms}$ \hspace{1cm} \textbf{Non-disruptive}

50 shots manually analyzed in populations $i$ and $iii$, confirmed that:

- \textbf{No false positives} in major disruptions (i.e. calling non-disruptive shot disruptive)
- \textbf{No false negatives} in non-disruptions (i.e. calling disruptive shot non-disruptive)
Assuming \( \frac{dj}{dr} \) monotonically decreasing, and \( q \) monotonically increasing, potential energy can be expressed as follows [Sykes PRL 80],

\[
\delta W = - \int_{0}^{r_{q2-w/2}} \left| \frac{dj}{dr} B_{r}^{2} r \right| dr + \int_{r_{q2+w/2}}^{a} \left| \frac{dj}{dr} B_{r}^{2} r \right| dr
\]

- Recall \( l_{i}/q_{95} \approx \alpha \rho_{q2} + c \), and therefore \( l_{i}/q_{95} \) determine limits of integration
- As \( l_{i} \) determines profile peaking, and \( q_{95} \approx 1/I_{p} \), \( dj/dr \) is expected to depend on \( l_{i}/q_{95} \)
$d_{\text{edge}}$ might be related to the physics of the thermal quench; other works that have also observed this...

- Experimental results from Compass-C find locked mode disruptions occur when inequality that is similar to $d_{\text{edge}}$ is satisfied [Hender NF 92]

$$w/(a - r_{q2}) > 0.7$$

- Massive gas injection simulations using NIMROD find the thermal quench is triggered when $m/n=2/1$ island intersects the radiating edge [Izzo NF 06]

- Stochastic layer exists inside the unperturbed LCFS [Evans PoP 02, Izzo NF 08], which could stochastize the $m/n=2/1$ island when $d_{\text{edge}}$ sufficiently small
Analytic current filament model used to map from $B_R$ at the external saddle loops (ESLDs) to perturbed current $\delta I$

- Informed by $n=1$ signal from ESLDs and $r_{q2}$ and $R$ from EFIT constrained by motional stark effect

- Find the following equation for $\delta I$,

$$\delta I = \frac{B_R^{n1}}{\pi \alpha(R) r_{q2}^2}$$

where $\alpha(R)$ is a quadratic function of $R$
Definition of $\delta I$, and mapping to island width $w$

- $\delta I_h$ assumes that a deficit of current exists in O-point, and excess in X-point
- $\delta I$ assumes that there is only current deficit, which is greatest at the O-point
- Note that both $\delta I$ and $\delta I_h$ produce the same non-axisymmetric field

Where $\delta I_h$ is in Amps, $r_{q2}$, $R$, and $dq/dr$ are in meters, $B_T$ is in Tesla, and $c=8/15$
View of $q=2$ surface during a disruption

- Estimated position and width of islands appear reasonable
\( \rho_{q2} \) increases through lifetime of IRLM

- Evolution of \( \rho_{q2} \) from locking to 100 ms before mode end
- Majority of disruptive IRLM move outwards
Modified Rutherford Equation (MRE)

\[ 0 = r \Delta'(w) + \alpha \varepsilon^{1/2} \frac{L_q}{L_{pe}} \beta_{pe} \frac{r}{w} + 4 \left( \frac{w_v}{w} \right)^2 \]

\[ (w_v/w)^2 \approx a w_v / w - b \]

\[ \frac{2w_{sat}}{r} = \left( \frac{C_0 - 4b}{C_1} \right) + \left[ \left( \frac{C_0 - 4b}{C_1} \right)^2 + \frac{4}{C_1} \left( \alpha \varepsilon^{1/2} \frac{L_q}{L_p} \beta_p + 4a \frac{w_v}{r} \right) \right]^{1/2} \]
LM disruptivity increases above density limit

- Disruptivity strongly increases past JET density limit