Local measurement of resonant error field using tearing mode dynamics in EXTRAP-T2R

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22.44 22.46 22.48 22.5 22.52
0 0.5 1
| \dot{B}_p | (arb)

\phi_{RM P} (\pi)
\phi_{measured} (\pi)

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Nov. 22, 2015
Outline

1. Motivation
2. EXTRAP-T2R and its diagnostics
3. Physical model of tearing mode (TM) interaction with error field (EF)
4. Experiments
5. EF phase identification
6. Conclusion
Motivation

1. Identification and correction of EFs is important to prevent tearing modes
2. Tearing response depends on resonant field local to the rational surface

Related work:

▶ "Resonant magnetic perturbation effect on tearing mode dynamics" - L. Frassinetti NF 2010
▶ "Error field assessment from driven rotation of stable external kinks at EXTRAP-T2R reversed field pinch" - F. Volpe NF 2013
▶ "Measurements of the toroidal torque balance of error field penetration locked modes" - D. Shiraki PPCF 2014

Novelty of this work:

▶ Use of fast rotating (natural frequency) tearing modes to diagnose error fields
Plasma properties:

- $R = 1.24$ m
- $a = 0.183$ m
- $I_p = 80 - 100$ kA
- $T_e \approx 300 - 400$ eV
- $n_e \approx 10^{19}$ m$^{-3}$

$m/n = 1/ -12$ tearing mode:

- Innermost tearing mode
- Co-rotates with plasma at $\sim 5$-10 kHz
- $\tau_R = \mu_0 r_s^2 \sigma(r_s) \approx 1$ ms

[Frassinetti NF 2010]

- $\tau_s = \tau_R \frac{\delta_s}{r_s}$

- Define $2\pi$ period phase, $12\phi \equiv \phi^{n=12}$

[Figure from L. Frassinetti NF 2014]
Diagnostics on EXTRAP-T2R

**Poloidal sensors:**

- 4 toroidal arrays of 64 probes spaced evenly poloidally and toroidally
- Sensors located outside of vessel with resistive time \( \tau_v \approx 0.25 \text{ ms} \) for \( m/n = 1/_{-12} \) [L. Frassinetti NF 2010]
- Sample rate = 1 MHz

**Control coils:**

- Feedback suppresses unstable modes at \( f_{\text{feedback}} = 10 \text{ kHz} \) with \( 4 \times 32 \) actuating coils outside of copper shells with \( \tau_w \approx 8 \text{ ms} \) for \( m/n = 1/_{-12} \) fields

*Image rendered by K.E.J. Olofsson*
Physical model for TM/EF interaction

1. Width modulation
2. Velocity modulation
1) Width modulation driven by EF at $\omega_o$

Assuming $\Delta'(w) = C_0/r_s - C_1/r_s^2$ [R. La Haye NF 2015], the first order width evolution of a classical, saturated, rotating TM in the presence of an EF [R. Fitzpatrick NF 1993] is given by,

$$\frac{\partial W_1}{\partial t} \approx -\frac{C_1}{r\tau_R} W_1 + \frac{2mr}{\tau_r} \left( \frac{W_v}{W_o} \right)^2 \cos(\Delta \phi)$$
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Assume uniform rotation, and that the TM and EF are aligned at $t = 0$ (i.e. $\Delta \phi = \omega_o t$),

$$W_1(t) = \frac{2mr}{\tau_R} \left( \frac{W_v}{W_o} \right)^2 \frac{C_1/(r\tau_R) \cos(\omega_o t) + \omega_o \sin(\omega_o t)}{\omega_o^2 + C_1^2/(r\tau_R)^2}$$

We expect $\omega_o \gg C_1/(r\tau_R)$, and therefore to first order we have,
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We expect $\omega_o \gg C_1/(r \tau_R)$, and therefore to first order we have,

$$W(t) = W_o + \frac{\delta W}{\omega_o \tau_R} \sin(\omega_o t)$$

To first order in $\delta W/W_o$,

$$b_p \approx b_{p0} + \delta b_p \sin(\omega_o t)$$

Local measurement of resonant error field using tearing mode dynamics in EXTRAP-T2R
2) Velocity modulation due to EF torque at $\omega_o$

- Simplest torque balance model*:

\[
I \ddot{\phi}_{tm} = -T_{em} \sin(\phi_{tm} - \phi_{ef}) + \nu(\dot{\phi}_o - \dot{\phi}_{tm})
\]

<table>
<thead>
<tr>
<th>Inertial torque</th>
<th>Error field torque</th>
<th>Viscous torque</th>
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*See L. Frassinetti NF 2010 for more detailed model
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Inertial torque    Error field torque    Viscous torque

- Assume the EF torque is order $\epsilon$, and use a perturbation expansion,

\[
\phi_{tm}(t) = \phi_0(t) + \epsilon \phi_1(t)
\]

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- Assume the EF torque is order $\epsilon$, and use a perturbation expansion,

\[
\phi_{tm}(t) = \phi_0(t) + \epsilon \phi_1(t)
\]

- Omitting exponentially decaying solutions, and setting $\phi_{ef} = 0$ (all phases relative to EF), the angular frequency to 1st order is,

\[
\dot{\phi}_{tm}(t) = \omega_o + \omega_v^m \left[ \cos(\omega_o t) - \frac{\omega_v}{\omega_o} \sin(\omega_o t) \right]
\]

\[
\omega_v^m = \frac{\omega_{em}/\omega_o}{1 + (\omega_v/\omega_o)^2} \quad \omega_{em} = \sqrt{\frac{T_{em}}{I}} \quad \omega_v = \frac{\nu}{I}
\]

*See L. Frassinetti NF 2010 for more detailed model
Deriving measured signals from model

Represent the tearing mode by a phasor,

$$B_p(t) = b_p(t) [\cos(\phi_p) \hat{x} + \sin(\phi_p) \hat{y}]$$

The magnitude of the time-derivative to first order in $\delta b_p / b_p$,

$$|\dot{B}_p(\phi)| = \dot{\phi}_p b_p \sqrt{1 + 2 \frac{\delta b_p}{b_p} \sin(\phi)}$$
Deriving measured signals from model

Represent the tearing mode by a phasor,

\[ \mathbf{B}_p(t) = b_p(t) \left[ \cos(\phi_p)\mathbf{\hat{x}} + \sin(\phi_p)\mathbf{\hat{y}} \right] \]

The magnitude of the time-derivative to first order in \( \delta b_p / b_p \),

\[ |\dot{\mathbf{B}}_p(\phi)| = \dot{\phi} b_p \sqrt{1 + 2\frac{\delta b_p}{b_p} \sin(\phi)} \]

Taking \( \sin(\phi) \to \sin(\omega_o t) \), and substituting \( \dot{\phi}(t) \),

\[ \left| \frac{\dot{\mathbf{B}}_p(t)}{\omega_o b_p} \right| \approx \left( 1 - \frac{\omega_v \omega_{vem} \delta b_p}{2\omega_o^2 b_p} \right) + \left( \frac{\delta b_p}{b_p} - \frac{\omega_v \omega_{vem}}{\omega_o^2} \right) \sin(\omega_o t) \]

\[ + \frac{\omega_{vem}}{\omega_o} \cos(\omega_o t) + \frac{\omega_{vem} \delta b_p}{2\omega_o b_p} \left[ \sin(2\omega_o t) + \frac{\omega_v}{\omega_o} \cos(2\omega_o t) \right] \]

and its angular position,

\[ \alpha = \tan^{-1} \left( \frac{\mathbf{B}_p \cdot \mathbf{\hat{y}}}{\mathbf{B}_p \cdot \mathbf{\hat{x}}} \right) \approx \phi \]
Experimental phase identification of proxy error fields using tearing mode dynamics
Experiment - diagnosing applied RMP via tearing mode dynamics

1. Suppress all tearing modes except the $m/n = 1/ - 12$ via feedback

2. Apply a static resonant magnetic perturbation (RMP) with magnitude $B_{RMP}$ and phase $\phi_{RMP}$

3. Observe the AC modulation in island width $W$ and frequency $\omega_{tm}$

4. Use characteristics of these modulations to measure the RMP applied in step #2
\(|\dot{B}_p\) exhibits dynamics with frequencies lower, higher, and equal to \(\omega_o\)
High-pass filtering $|\dot{B}_p|$ isolates the signal of interest

- Inter-period mode dynamics not studied here
- Sub-period modulation is expected to be driven by RMP
- $2\omega_o$ response is visible

Local measurement of resonant error field using tearing mode dynamics in EXTRAP-T2R
Detect "suitable" time intervals $\tau_i$ for analysis

Suitable means:

- Full rotation
- $f > 20$ kHz
- $\tau_i > 0.1$ ms
Identify $\phi$ when $\delta |\dot{B}_p|$ is maximized in each rotation period.

1. Within each period, locate the maximum $\delta \dot{B}_p$. 

![Graph showing time vs. $\delta dB/dt$ and $\phi_{lm}$](image_url)
Identify $\phi$ when $\delta|\dot{B}_p|$ is maximized in each rotation period.

1. Within each period, locate the maximum $\delta\dot{B}_p$.
2. Find the corresponding $\phi \equiv \phi_{\text{max}}$. 

Local measurement of resonant error field using tearing mode dynamics in EXTRAP-T2R
Identify $\phi$ when $\delta |\dot{B}_p|$ is maximized in each rotation period

1. Within each period, locate the maximum $\delta \dot{B}_p$
2. Find the corresponding $\phi \equiv \phi_{max}$
3. Save $\phi_{max}$ for each period and produce histogram for shot
Polar histograms of \( \max(\dot{B}_p) \) show phase preference

- 9 of these paneled on next slide
- Expect agreement between centroid and RMP phase
Toroidal scan of 2 G RMP shows that centroid technique identifies phase of RMP

\[ \phi_{\text{RMP}} = 0 \]

\[ \phi_{\text{RMP}} = \frac{\pi}{4} \]

\[ \phi_{\text{RMP}} = \frac{2\pi}{4} \]

\[ \phi_{\text{RMP}} = \frac{3\pi}{4} \]

\[ \phi_{\text{RMP}} = \frac{\pi}{1.18} \]

\[ \phi_{\text{RMP}} = \frac{5\pi}{4} \]

\[ \phi_{\text{RMP}} = \frac{6\pi}{4} \]

\[ \phi_{\text{RMP}} = \frac{7\pi}{4} \]

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\[ f_{cc} \cos(\phi_{RMP}) \text{ (arb)} \]
\[ f_{cc} \sin(\phi_{RMP}) \text{ (arb)} \]

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Local measurement of resonant error field using tearing mode dynamics in EXTRAP-T2R
Measured RMP phases in good agreement with actual RMP phases

- $\sqrt{\sum \text{(errorbars)}^2} \rightarrow$ phase resolution better than ±30 degrees

- Trend of leading in quadrants 1 and 2, lagging in quadrants 3 and 4

Local measurement of resonant error field using tearing mode dynamics in EXTRAP-T2R
Residual model might explain leading and lagging behavior of measured RMP

- Assuming the total field is the sum of an intrinsic EF and the applied RMP,

\[
\phi_{\text{meas}} = \tan^{-1} \left( \frac{B_{\text{RMP}} \sin(\phi_{\text{RMP}}) + B_{\text{ef}} \sin(\phi_{\text{ef}})}{B_{\text{RMP}} \cos(\phi_{\text{RMP}}) + B_{\text{ef}} \cos(\phi_{\text{ef}})} \right) + \delta \phi
\]

Inferred error field:
\[
\phi_{\text{ef}} \approx -20^\circ \\
B_{\text{ef}} \approx 1 \text{ G}
\]
Conclusions

1. The phase of an applied RMP can be extracted with at least $\pm 30^\circ$ accuracy from $\dot{B}_p$, apart from an $m = 1$ discrepancy which might be related to the intrinsic machine error.

2. Response of the tearing mode at $2\omega_o$ has been observed in $\dot{B}_p$ data, and modeling suggests this is evidence of modulation in both island velocity and amplitude at $\omega_o$.

Future work:

1. Extract RMP amplitude from tearing mode dynamics
2. Assess how physics scale to different machines, including ITER.
Frassinetti NF 2012, \( R = 1.24 \) m, \( a = 0.183 \) m, \( I_p = 80 – 100 \) kA, \( T_e = 300 – 400 \) eV, \( n_e = 10^{19} \) m\(^{-3}\), \( r_v = 0.192 \) m, \( \tau_v = 0.25 \) ms for \( m/n = 1/ – 12 \), \( \tau_w = 8 \) ms for \( m/n = 1/ – 12 \), \( \nu_{kin} = 1 – 40 \) m\(^2\)s\(^{-1}\), \( B_\theta^0 \approx 0.12 \) T and \( |B| = 0.18 \) T at \( r/a = 0.4 \)

Frassinetti NF 2010, \( \tau_v = 0.28 \) ms for \( m/n = 1/ – 12 \), wall torque not significant, \( \omega = 20 – 50 \) krad s\(^{-1}\), \( \rho(r) = \rho_0(1 – r^3/a^3) \) with \( \rho_0 = 1.5 \times 10^{-8} \) kg m\(^{-3}\), \( \nu(r) = \nu_0(1 + r^2/a^2) \), \( \nu_0 = 0.65 \times 10^{-7} \) kg ms\(^{-1}\), \( \nu = \mu/\rho \), \( \omega \tau_v = 5.5 – 14 \), \( r_s/a = 7\% \) which for \( a = 0.183 \) m gives \( r_s = 0.013 \) m, \( \tau_R = \mu_0 r_s^2 \sigma(r_s) = 1.1 \) ms, \( \tau_V = 2600 \) s, \( \tau_H = 1.6 \times 10^{-6} \) s

Frassinetti PPCF 2014, \( s = (r/q)(dq/dr) = -0.07 \)
\[ \eta = \frac{\pi Z e^2 m_e^{1/2} \ln(\Lambda)}{(4\pi \epsilon_0)^2 (k_B T_e)^{3/2}} = 1.86 \times 10^{-7} \Omega \cdot m \]

\[ \sigma = \frac{1}{\eta} = 5.38 \times 10^6 S/m \]
The maxima of \( B_r \) and \( B_p \), and therefore \( \phi_r \) and \( \phi_p \) differ by \( \pm \pi/2 \). The sign depends on the helicity of the plasma. Without defining the helicity, we use \( S_x \) and \( S_y \) to represent the sign on these terms. Note that due to rotation in the \( +\hat{\phi} \), we know that \( S_x S_y = -1 \).

\[
B_p(t) = b_p(t) [S_x \sin(\phi_r) \hat{x} + S_y \cos(\phi_r) \hat{y}]
\]

The \( \dot{B}_p \) phasor,

\[
\dot{B}_p(\phi) = \left[ \dot{b}_p(t) S_y \cos(\phi_r) - b_p(t) S_y \dot{\phi}_r \sin(\phi_r) \right] \hat{y} + \left[ \dot{b}_p(t) S_x \sin(\phi) + b_p(t) S_x \dot{\phi}_r \cos(\phi_r) \right] \hat{x}
\]

Keeping only leading order terms, the phase of \( \dot{B}_p \) is therefore given by,

\[
\alpha = \tan^{-1}\left( \frac{\dot{B}_p \cdot \hat{y}}{\dot{B}_p \cdot \hat{x}} \right) \approx \tan^{-1}\left( \frac{-b_p S_y \dot{\phi}_r \sin(\phi_r)}{b_p S_x \dot{\phi}_r \cos(\phi_r)} \right) \approx \phi_r
\]

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Tearing mode responds to EF torque viscously

\[ v_\phi \text{ peaks when mode in phase quadrature with RMP} \]

Recall:

\[ \dot{\phi}_{tm}(t) = \omega_o + \omega_v^{em} \left[ \cos(\omega_o t) - \frac{\omega_v}{\omega_o} \sin(\omega_o t) \right] \]

• Suggests \( \omega_v / \omega_o \gg 1 \)

[figure from L. Frassinetti NF 2012]

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