Inverse methods for stellarator error fields and emission

Ken Hammond
Columbia University

A. Anichowski
P. W. Brenner
R. R. Diaz-Pacheco
S. Lazerson (PPPL)
S. D. Massidda (Auburn)

T. S. Pedersen (IPP)
S. Raftopoulos (PPPL)
P. Traverso (Auburn)
F. A. Volpe
Y. Wei
CNT: an introduction

IL coils

PF coils

1 m
Outline

Error field diagnosis

Reconstruction of emissivity profiles
Error field diagnosis
Initial agreement between measured and predicted flux surfaces was poor

\[ I_{IL}/I_{PF} = 3.68 \]

\[ I_{IL}/I_{PF} = 3.18 \]
Objective: deduce coil misalignments based on observed Poincaré cross-sections

\[
X(p) \text{ can be determined with a field line tracer}
\]

Finding \( p \) for a given \( X \) requires an iterative method
Use Newton-Raphson algorithm to find $p^*$ such that $X(p^*) = X^*$

Discrepancy: $F(p_0) = X(p_0) - X^*$

Improve guess with step $\delta p$ satisfying $F = -J \delta p$

- Jacobian: $J_{ij} = \partial F_i / \partial p_j$
- Best linear unbiased estimator for $\delta p$: $(J^T C^{-1} J)^{-1} J^T C^{-1} F$
- $\delta p_1 \rightarrow p_1 = p_0 + \delta p_1 \rightarrow X(p_1) \rightarrow F(p_1) \rightarrow \delta p_2 \rightarrow \ldots$

Minimize: $\chi^2 = \frac{1}{2} F \cdot C^{-1} F$
IL coil displacements optimized to fit experimental data for $I_{IL}/I_{PF} = 3.68$

- IL coils each allowed 5 degrees of freedom
- Reduction of $\chi^2$ by factor $> 100$
- Principal misalignments:
  - Both moved ~2 cm in -y direction
  - One had a tilt offset of ~1°
Cross-sections for optimized parameters agree better with measurements.

$\frac{I_{IL}}{I_{PF}} = 3.68$

$\frac{I_{IL}}{I_{PF}} = 3.18$
Reconstruction of emissivity profiles
Onion-peeling technique infers emissivity profile from a single camera image

- Assumptions:
  - uniformly emitting layers \( (e_j) \)
  - layers contribute to brightness \( (p_i) \) in proportion to sightline path length \( (L_{ij}) \)

- \( b_p = L e \)

- Pseudo-invert to deduce profile:
  \[
  e = (L^T C^{-1} L)^{-1} L^T C^{-1} b_p
  \]
Testing the concept by illuminating single layers with a movable hot cathode

$r = 2.3 \text{ cm}$

$r = 3.3 \text{ cm}$

$r = 5.7 \text{ cm}$
Reconstructed profiles of glow discharges exhibit expected features.

![Graph showing emissivity vs. effective minor radius for different minor radii (2.3 cm, 3.3 cm, 5.7 cm).](chart.png)
Profile of a <1 kW electron-cyclotron-heated discharge

- Hollow as expected
- High value on edge may result from unaccounted-for scrape-off layer
Conclusions

- Error field diagnosis inferred stellarator coil misalignments
- Onion-peeling inversion obtained 1D emissivity of 3D plasmas
- Further reading:
Future work

Error field diagnosis
- Additional degrees of freedom (more coils, coil deformation)
- Simultaneously use data from multiple current ratios
- Validation with high-precision metrology
- Identify vacuum configurations that mimic high-\(\beta\) configurations

Emissivity profiles
- Incorporate scrape-off layer in onion-peeling inversion
- Use with fast camera to monitor profile evolution during discharge

(Hölbe et al., NF 56, 2016)
Characterization of ECRH plasmas in the CNT stellarator

Ken Hammond

*Columbia University*

A. Anichowski  
P. W. Brenner  
R. Diaz-Pacheco  
S. Lazerson (PPPL)  
S. D. Massidda (Auburn)  
T. S. Pedersen (IPP)  
S. Raftopoulos (PPPL)  
R. Raman (PPPL)  
P. Traverso (Auburn)  
F. A. Volpe  
Y. Wei
Flux surfaces are measured with an electron beam and a phosphor-coated rod.

Long-exposure photograph

Electron gun

Plane for cross-sections
- Markers placed on chamber and coils
- Software generates point cloud from relative positions of markers

Result: PF coils as much as 40 mm off nominal position in some places
Supplement: X vector consists of discrete parameters characterizing the Poincaré cross-section

- Fourier series in $\theta$ for each flux surface

\[ R(\rho, \theta) = R_0(\rho) + \sum_{m=1}^{M} R_{cm}(\rho) \cos(m\theta) + \sum_{m=1}^{M} R_{sm}(\rho) \sin(m\theta) \]

\[ Z(\rho, \theta) = Z_0(\rho) + \sum_{m=1}^{M} Z_{cm}(\rho) \cos(m\theta) + \sum_{m=1}^{M} Z_{sm}(\rho) \sin(m\theta) \]

- Polynomial expansion in $\rho$ for each Fourier coefficient

\[ R_{c1}(\rho) = R_{c10}P_0(\rho) + R_{c11}P_1(\rho) + \ldots + R_{c1S}P_S(\rho) + \ldots \]

\[
\begin{array}{c|c}
 s & P_s(\rho) \\
 \hline
 0 & 1 \\
 1 & \sqrt{3} (2\rho - 1) \\
 2 & \sqrt{5} (6\rho^2 - 6\rho + 1) \\
 3 & \sqrt{7} (20\rho^3 - 30\rho^2 + 12\rho - 1) \\
 4 & 3 (70\rho^4 - 140\rho^3 + 90\rho^2 - 20\rho + 1) \\
\end{array}
\]

\[ \mathbf{x} = \begin{pmatrix} \vdots \\ R_{c10} \\ R_{c11} \\ \vdots \end{pmatrix} \]
Supplement: first verification allowed only two degrees of freedom

- $X^*$ calculated for $-0.45^\circ$ change in tilt angle
- Correct solution obtained in 4 steps
Supplement: coil displacement parameters

\[ p = \{ x_{IL1}, y_{IL1}, z_{IL1}, a_{IL1}, b_{IL1}, x_{IL2}, y_{IL2}, z_{IL2}, a_{IL2}, b_{IL2} \} \]
Supplement: verifications with 10 free coil parameters

Target: $z_{IL1} = 5$ mm

Target: $b_{IL1} = 0.002; b_{IL2} = 0.004$
Supplement: experimental conditions that pose challenges for island characterization

Cross-sections cut off due to rod extent

Shadowing of beam near rational surfaces
Supplement: inferred coil positions support observed $\tau = 1/3$ rational surfaces

(a) Nominal coil positions

(b) Inferred coil positions