Dynamics, stability and stabilization of magnetic islands by feedback phase-control and synchronized ECCD

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with

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Outline

• Introduction
  – Simulation of magnetic island dynamics

• Experimental results of feedback phase control
  – Mode phase control using RMPs in feedback

• Study of island stabilization
  – Mode amplitude control with synchronized ECCD

• Future work
  – improvements to controller
• **Introduction**
  – Simulation of magnetic island dynamics

• **Experimental results of feedback phase control**
  – Mode *phase* control using RMPs in feedback

• **Study of island stabilization**
  – Mode *amplitude* control with synchronized ECCD

• **Future work**
  – Improvements to controller
Modeling fixed-width mode dynamics under influence of torques

\[ I \frac{d^2 \phi}{dt^2} = T_{wall} + T_{EF} + T_{RMP} + T_{TM} + T_{visc} + T_{NBI} \]

E.M. Torques on Island

Non-E.M. Torques

2/1 magnetic island

I-coils

C-coils
Only EM torques are included in simulation

\[ \frac{I \, d^2 \phi}{dt^2} = T_{\text{wall}} + T_{\text{EF}} + T_{\text{RMP}} + T_{\text{TM}} + T_{\text{visc}} + T_{\text{NBI}} \]

- **Neglected torques include:**
  - Torque from other tearing modes, if no other modes exists
  - Viscous torque, if plasma rotation is low near the rational surface
  - NBI torque, if power is low or injection is balanced

![Graph showing DIII-D Torques](image)

**DIII-D Torques**
- Steady State wall torque
- Total NBI torque
Critical steady entrainment frequency depends on island width and coil current

\[ 0 = T_{\text{wall}} + T_{\text{RMP}} \]

- Max frequency at which smooth entrainment is possible
  - increases with coil current
  - decreases with island width

K.E.J. Olofsson PPCF 2016
Model adapted to ITER, with following assumptions

- **coils:**
  - External coils: 3 sets of 6
  - Internal coils: 3 sets of 9

- **major radius: 6.2 m**
- **wall time: 188 ms**
- **density: $7.2 \times 10^{19} \text{ m}^{-3}$**

ITER treated with 2 walls:
1) tiled Be first wall
2) vacuum vessels

![Diagram of ITER model](image)

- **Combined VV**
- **First Wall**
- **Separatrix**
- **q=2 surface**
- **Mag Axis**

![Diagram of Major Radius vs. Height](image)
Static applied RMP simulation on ITER show island alignment

- Simulation for
  - 5 cm island
  - 10 kA/turn in external coils at different phase
  - 1 G error field at 0°

- Island aligns with net of applied RMP and EF
Successful 5 Hz preemptive entrainment on ITER

- "Preemptive" entrainment means to apply rotating RMP before island locking
  - shown on DIII-D to prevent locking
- Simulation for
  - 5 cm island
  - 10 kA/turn in external coils rotating at 5 Hz
Small (< 5cm) ITER 2/1 mode can be entrained at low frequency

- Again, look for torque balance between $T_{\text{wall}}$ and $T_{\text{RMP}}$
  - assumed well-corrected EF
- $T_{\text{wall}}$ on island for steady state

![Graph showing ITER Steady State Wall Torque and ITER Critical Entrainment Frequency](image)
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Proportional-integral controller for mode phase has been implemented at DIII-D

\[ \Phi_{\text{error}} = \Phi_{\text{ref}} - \Phi_{\text{mode}} \]

\[ \Phi_{\text{correction}} = \text{PI control} \left( \Phi_{\text{error}} \right) \]

\( \Phi_{\text{reference}} \) requested

\( \Phi_{\text{mode}} \) from magnetic signals

reference

mode
Proportional-integral controller for mode phase has been implemented at DIII-D

\[ \Phi_{\text{error}} = \Phi_{\text{ref}} - \Phi_{\text{mode}} \]

\[ \Phi_{\text{correction}} = \text{PI control} (\Phi_{\text{error}}) \]

\[ \Phi_{\text{RMP}} = \Phi_{\text{mode}} + \Phi_{\text{corr.}} \]

Limit to \( \Phi_{\text{mode}} \pm 90^\circ \) for max torque

\[ \Phi_{\text{corr.}} \]
Proportional-integral controller for mode phase has been implemented at DIII-D

\[ \Phi_{\text{error}} = \Phi_{\text{ref}} - \Phi_{\text{mode}} \]

\[ \Phi_{\text{correction}} = \text{PI control}(\Phi_{\text{error}}) \]

\[ \Phi_{\text{RMP}} = \Phi_{\text{mode}} + \Phi_{\text{corr.}} \]

Limit to \( \Phi_{\text{mode}} \pm 90^\circ \) for max torque

Applied to coils
Controller successfully tracked reference phase

- tested both fixed phase and rotating reference
Controller successfully tracked reference phase

- tested both fixed phase and rotating reference
- Error decrease with increasing gain as expected
Simulated tracking error is dependent on gain, rotation frequency

- For P-only control, tracking error \(< \phi_{\text{error}} >\) increases with:
  - decreasing gain
  - increasing rotation frequency, island width, minor radius

![Graphs showing the relationship between tracking error and proportional gain, rotation frequency, and parameters like island width and minor radius.](image_url)
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ECCD deposition in physical coordinates, and spreading over flux surface

For duty cycle $\tau = 0.7$

O-point alignment

current drive FWHM $w_{ECCD} = \text{island full width}$

$J_{ECCD}(r, \Delta R) = J_{CDexp} \left( -\frac{(r - r_s - \Delta R)^2}{2(w_{ECCD}/2.355)^2} \right)$

- Solid colors show deposition timing and location
- Lighter colors depict current drive spreading over flux surfaces
- Green/red color is only a model of effect of ECCD, deposition just inside separatrix actually has negative impact
Time dependent of effect of ECCD was modeled using thin slices

\[
\frac{\tau_R}{r} \frac{dw}{dt} = \Delta' + a_2 \frac{j_{bs}}{j_{||}} \frac{L_q}{w} \left[ 1 - \frac{w_{\text{marg}}}{3w^2} - K_1 \left( \frac{w_{cd}}{w_{cd}} \frac{\Delta R}{\Delta \Phi} \frac{j_{cd}}{j_{bs}} \right) \right]
\]

- ECCD affects the modified Rutherford equation\(^1\) (MRE) most strongly through current replacement term
  - calculate effect of a blip of current deposited instantaneously
  - can predict sub-period effect of ECCD

1. La Haye PoP 2006
Time dependent of effect of ECCD was modeled using thin slices

\[ \frac{\tau R}{\tau} \frac{dw}{dt} = \Delta' r + a_2 \frac{j_{bs}}{j_{||}} L_q \left[ 1 - \frac{w_{marg}^2}{3w^2} - K_1 \left( w_{cd}, \frac{\Delta R}{w_{cd}}, \Delta \Phi \right) \frac{j_{cd}}{j_{bs}} \right] \]

- Instantaneous values of efficiency $K_1$ can be calculated

![Diagram showing ECCD Deposition and Efficiency $K_1$ vs Helical Angle $\xi$]
Well-aligned, modulated deposition in O-point is most efficient.

- Efficiency contours as a function of
  - radial misalignment
  - island size
- O-point modulated ECCD\(^1\) has higher efficiency
  - more forgiving in terms of radial misalignment
- X-point modulation exacerbates the mode
  - negative effect is diminished as misalignment increases

1. Perkins EPS 1997
Radial (mis)alignment is also important for island suppression

- Experimentally observed that nominal O-point deposition of current increased the island size

- Analysis (ray-tracing, equilibrium fitting, and magnetics) puts ECCD deposition profile outside island separatrix
O-point deposition increase mode size explained by radial misalignment

- Compared actual trajectory with MRE predictions
- Well-aligned ECCD is expected to completely suppress island
Radially misaligned X-point deposition stabilizing island is still being investigated

- Shot-to-shot phase scan was performed during same day
  - found a stabilizing phase for fixed radial location
- Deposition at X-point toroidally, outside island radially, resulted in amplitude suppression
- Suggests other physics are involved?
Conclusions

- A numerical model correctly simulates mode behaviour for DIII-D.
  - When adapted to ITER, it predicts entrainment of small (5 cm) islands in sub-10 Hz range

- A feedback controller has been implemented on DIII-D, able to prescribe fixed phase and rotating reference

- Precise control of mode phase allows synchronization with ECCD for mode suppression.
  - Experimental results qualitatively agree with radially misaligned model
Future work

• Upgrades to the controller are currently underway
  – Real-time calculation of frequency to compensate phase lag in magnetic measurement
  – Allows for better control of mode position
  – Expect to be able to entrain at higher frequency (above inverse wall time ~50 Hz)
Back up slides
Simulation of mode dynamics with some simplifying assumptions

- **Simplified equation of motion**
  \[
  I \frac{d^2 \phi}{dt^2} = T_{\text{wall}} + T_{\text{EF}} + T_{\text{RMP}}
  \]

- **Condition for smooth entrainment**
  \[
  0 = T_{\text{wall}} + T_{\text{RMP}}
  \]

\[
T_{\text{wall}} = -\frac{2\pi R \left[ B_R(b) r_{mn} \right]^2}{\mu_0 b} \left[ \frac{r_{mn}}{b} \right]^{2m-1} \frac{\Omega_T}{1 + (\Omega_T)^2}
\]

\[
T_{\text{EF}} = -\pi^2 R^2 m \frac{a}{r_{mn}} I_{\text{EF}} B_R(a) \sin[n\phi(t)]
\]

\[
T_{\text{RMP}} = -\pi^2 R^2 m \frac{b}{r_{mn}} I_{\text{RMP}} B_R(b) \sin[n\phi(t) - n\phi_{\text{RMP}}(t)]
\]
Define non-dimensional helical flux $\Psi$, approximates distance from island separatrix

$$\Psi = \frac{(r - r_s)^2}{w_{island}^2} - \left(1 + \frac{\cos(m\xi)}{2}\right)$$

$r_s = \text{rational q surface}$

$w_{island} = \text{island half-width}$

helical angle $\xi = \theta - \frac{n\Phi}{m}$

$-\pi \leq \xi < \pi$

Contour of $\Psi$

$r = \text{rational q surface}$

$w_{island} = \text{island half-width}$

$
\begin{align*}
\Psi &= -1 \text{ island center} \\
\Psi &= 0 \text{ island separatrix} \\
\Psi &> 0 \text{ outside island} \\
\text{at } \xi = 0, \Psi \text{ from 0 to 1 corresponds to } 0.4 \times w_{island}
\end{align*}$
ECCD deposition in physical coordinates, and spreading over flux surface

For duty cycle $\tau=0.7$, Phase misalignment $\Delta \Phi=0$ (O-point alignment), and current drive FWHM $w_{ECCD}$ = island full width

$$j_{ECCD}(r, \Delta R) = J_{CDexp} \left( -\frac{(r - r_s - \Delta R)^2}{2(w_{ECCD}/2.355)^2} \right)$$

Solid colours show deposition timing and location, and lighter colours depict current drive spreading over flux surfaces
ECCD deposition in flux coordinates (O-point)

Current Distribution for various $\tau$

Current profile in flux coordinates calculated from physical coordinates

$$J(\Psi, x, \tau, \Delta \phi) = \frac{1}{Vr(\Psi)} \int d\xi V(\Psi, \xi) M(\xi, \tau, \Delta \phi) j(r, x, \xi)$$

$$M(\xi, \tau, \Delta \phi) = \begin{cases} 
1 & 0 < |m(\xi + \Delta \phi) + 2\pi l| \leq \tau \pi \\
0 & \tau \pi < |m(\xi + \Delta \phi) + 2\pi l| \leq \pi 
\end{cases}$$

Modulation term

For reference

$\tau = 0.5$

$w_{ECCD} = w_{island}$
ECCD deposition in flux coordinates, O- and X- points comparison

Current Distribution for various $\tau$

O-point

X-point

ECCD Deposition

Radial position $(r - r_s)/w_{island}$

Helical Angle $\xi$

ECCD Deposition

Radial position $(r - r_s)/w_{island}$

Helical Angle $\xi$
Effectiveness of current drive dependent on exact deposition location

\[ K_1(x, \tau, \Delta \phi) = \frac{1}{C_2} \int_{-1}^{\infty} d\Psi \, W(\Psi) \, J(\Psi, x, \tau, \Delta \phi) \]

\[ W(\Psi) = \int \frac{\cos(m\xi) d\xi}{\left[\Psi + \frac{1 + \cos(m\xi)}{2}\right]^{1/2}} \]

- Weight function \( W(\Psi) \) calculated from island evolution equation [Perkins]
- Highest weight at center of island
- Negative contributions near separatrix
Efficiency $K_1$ as a function of duty cycle, for different phasing

- **O-point and X-point solutions converge at $\tau=0$ and $\tau=1$**

- **Phasing is important, but forgiving**
  - only X-point deposition is destabilizing

- **Most efficient duty cycle is not necessarily 50%**
  - shape of curves dependent on other parameters: $w_{\text{island}}/w_{\text{ECCD}}$

Assumes good radial alignment, looking at effect of toroidal phasing

Assumes good radial alignment, looking at effect of toroidal phasing
Efficiency $K_1$ drops quickly if $\Delta R > 0.3w_{ECCD}$

- Also dependent on $w_{\text{island}}/w_{ECCD}$ ratio
- Decrease is more pronounced for most efficient duty cycle $\tau$
- All curves tend towards 0 at large misalignment
  - model only accounts for direct replacement of current
Corroborating results of deposition at other phases

(a) Nominal O–point Deposition
(b) Nominal X–point Deposition
(c) Nominal O– to X-point Transition
(d) Nominal X– to O-point Transition